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Predicting the neutrino-spectrum in SUSY-SO(10)

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ABSTRACT: We present a systematic search for SUSY-SO(10) models which predict the neutrino properties. The models are based on the five sets of quark mass matrices, with texture zeros, discussed recently by Roberts, Ramond and Ross. We found 8 such neutrino textures three of which can solve the solar neutrino problem. The latter have tau-neutrino masses of few eV i.e. relevant for cosmology and $\nu_\tau - \nu_\mu$ mixing angles that can be observed by the CHORUS, NOMAD and P803 experiments.

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1 Introduction

The recent strong evidence for the top quark in CDF [1] completed the information we have on the masses and mixing angles of the quarks. It emphasizes, however, at the same time our ignorance of their origin. As the fermionic masses are free parameters in the standard model (SM), an embedding into a grand-unified-theory (GUT) can help. This is also suggested by the unification of the gauge coupling constants of the SM [2], at 10^{16} GeV, provided the spectrum is extended into that of the minimal SUSY-SM (MSSM) [3].³ GUTs give relations between the Yukawa coupling constants of different flavours, like the successful $Y_\tau(GUT) \simeq Y_b(GUT)$ lepton-quark one. Yet, the complete understanding of the mass-mixing pattern requires relations between the families. This can come only from outside the GUT, by using a family-symmetry (or superstrings?). The only phenomenological indication in this direction is that the mixing angles and masses of the quarks are consistent with the appearance of texture zeros in the Yukawa matrices [5].

A recent study by Roberts, Ramond and Ross (RRR) [6] found five different sets of symmetric quark mass matrices with texture zeros, which account for the quark masses and mixing. Special examples, like the Fritzsch [7] texture, were known before. Also Dimopoulos, Hall and Rabi (DHR) [8] discussed in detail quark and charged leptons mass matrices suggested by Harvey, Ramond and Reiss [9], in terms of a SUSY- SO(10) broken directly into the MSSM.

All this is true for the quarks and charged leptons. The neutrino-masses and mixing are completely unknown. Except for possible experimental indications coming from the solar-neutrino-puzzle (SNP) [10], the depletion of the atmospheric ν_μ [11] and some cosmological dark matter arguments [12]. All of which are consistent with possible neutrino masses in the range of 10^{-5} eV – 3 eV .

Such small neutrino masses are obtained in L-R symmetric GUTs, like SO(10), using the see-saw mechanism. This means that the SU(5) singlet RH-neutrinos acquire large Majorana masses. The diagonalization of the complete neutrino mass matrix leads then to three small eigenvalues.

In a previous paper [13], we were able to predict the neutrino properties, by requiring that all matrices, including the RH-neutrino Majorana mass one, have the same Fritzsch-texture. The model was based on SUSY-SO(10) with the scale of the RH-neutrino mass matrix taken at the unification energy – as is natural in SUSY theories. It gives neutrino masses and mixing angles consistent with a possible solution of the solar neutrino puzzle, without the need for a free parameter. Unfortunately, if top was observed at CDF, its mass is too high to be consistent with such a model.

³ Another possibility is to introduce an intermediate breaking scale at $\approx 10^{12}$ GeV. [4]

In order to be able to predict the neutrino properties in terms of more complicate textures, we must use stronger assumptions. The RH-neutrino scale becomes then a free parameter and to solve the SNP, it must be lower then the GUT scale by several orders of magnitude. Assuming, as in almost all recent fermionic mass models, that the SUSY-GUT is broken directly into the MSSM, it is not clear where this intermediate mass scale is coming from.

In this paper we use SUSY-SO(10) with the three families of the quarks and the leptons, including ν_R , in the **16** representation Ψ_i , $i = 1, 2, 3$. In the view of the content of,

$$\mathbf{16} \times \mathbf{16} = (\mathbf{10} + \mathbf{126})_{\text{symmetric}} + \mathbf{120}_{\text{antisymm.}}$$

only the $\phi_{\mathbf{10}}$ and $\phi_{\overline{\mathbf{126}}}$ Higgs representations can contribute to the symmetric Yukawa terms.

The most general Yukawa Lagrangian at the GUT scale is then:

$$\mathcal{L}_Y = \sum \overline{\Psi}_i^c \Psi_j (Y_{ij}^{\mathbf{10}} \phi_{\mathbf{10}}^{ij} + Y_{ij}^{\overline{\mathbf{126}}} \phi_{\overline{\mathbf{126}}}^{ij}) . \quad (1)$$

Note, that one can absorb the difference between $Y_{ij}^{\mathbf{10}}$ and $Y_{ij}^{\overline{\mathbf{126}}}$ in the VEVs, and use only one effective Yukawa matrix, if all the Higgs representations are different. (This was used in the previous paper [13] but is not true here, as we shall see later).

Our aim is to predict the neutrino properties in terms of the mass matrices of the quarks and the charged leptons. In order to do this we shall use the requirements suggested by DHR [8]⁴ and apply them to the five texture sets of RRR.

The requirements combine actually *predictability* and *minimality* as follows:

1. The textures of the mass matrices are dictated by discrete symmetries and the directions of the VEVs in such a way that the minimal number of higgs multiplets is used.
2. Each fermion mass matrix element is generated by a VEV of only one of the $\phi_{\mathbf{10}}$ or $\phi_{\overline{\mathbf{126}}}$ multiplets.
3. All entries of the RH-neutrino Majorana mass matrix, M_{ν_R} , must be induced by one $\phi_{\overline{\mathbf{126}}}$ multiplet and in such a way the matrix is not singular.

⁴They used those requirements for “their” texture, which is very probably excluded experimentally as it requires $|V_{cb}| > 0.5$.

The textures of RRR do not tell us if the non-vanishing entries are due to the $\phi_{\mathbf{10}}$ or the $\phi_{\overline{\mathbf{126}}}$ Higgs representation. More information about M_d , the down quarks mass matrix, can be obtained using the “connection” between this matrix and that of the charged leptons, M_ℓ . In view of the fact that the $\phi_{\overline{\mathbf{126}}}$ contributions come with a relative Clebsch-Gordan coefficient of (-3), the fit of the M_ℓ elements to the lepton masses can tell us where $\phi_{\overline{\mathbf{126}}}$ contributes. It was already pointed out by Georgi and Jarlskog [14] that the Ansatz

$$m_\tau = m_b \quad m_\mu = 3m_s \quad m_e = 1/3m_d$$

at the GUT scale, works very well. This Ansatz can be generated by a factor (-3) in the $(M_\ell)_{22}$ matrix element [9] [8]. RRR checked it for their textures in terms of the SUSY-GUT broken directly into the MSSM [6]. This is obviously consistent with our requirement (1). Note, also that the leading (3,3) elements are generated in such a case by $\phi_{\mathbf{10}}$. Thus, they obey $(M_d)_{33} = (M_\ell)_{33}$, and one has in addition, the successful approximate (Yukawa) $Y_b(\text{GUT}) \simeq Y_\tau(\text{GUT})$ unification.

The structure of the mass matrix of the “up”-quarks, M_u , cannot be fixed using similar arguments, as the related neutrino Dirac mass matrix, M_{ν_D} , is phenomenologically unknown. However, we will show that our minimality and predictability requirements limit considerably the possibilities.

We found using our method eight sets of symmetric textures which predict the neutrino properties – up to the overall mass scale of the RH- neutrino masses. We evolved then the Yukawa matrices, from the GUT scale to low energies, using the renormalization group equations (RGEs). The resulting matrices are then fitted to the low-energy experimental data, and this fixes the quark parameters at the GUT scale. Those parameters dictate the entries of the light-neutrino parameters, for each one of the eight sets of textures. At the same time, we have also predictions for certain quantities in the quark-sector which we can used as a test. The light (see-saw) neutrino mass matrix is then evolved to low energies.

The resulting neutrino properties are given in terms of their mass ratios and mixing angles. The absolute neutrino masses can be obtained only when the intermediate scale, relevant for the overall scale of the RH- neutrino mass matrix, is given.

The mixing angles of all the eight texture sets are such that $\sin^2 2\theta < 0.2$, and hence, we cannot have vacuum oscillation as a solution to the solar neutrino puzzle. Also, the possible depletion of the atmospheric ν_μ cannot be accounted for. The values of the $\nu_e - \nu_\mu$ mixing angle (i.e. $\sin^2 2\theta_{e\mu}$) are generally in the range of the small angle (i.e adiabatic) MSW [15] solution to the SNP. Requiring that $\Delta m_{e\mu}^2$ has the right value for this solution, we obtain for the RH-neutrino scale:

$$M_R \sim 10^{13} - 10^{14} \text{GeV}.$$

The corresponding masses of ν_τ are of few eV – in the range interesting for cosmology [12]. At the same time the values of $\sin^2 2\theta_{\mu\tau}$ are such that ν_τ oscillations will be observed in experiments like CHORUS [16], NOMAD [17] and P803 [18].

The plan of the paper is as follows. In sect. 2 the models (and their discrete symmetries) will be discussed in detail. Sect. 3 will explain the details of our numerical analysis. In sect. 4 we will give and discuss the results in the neutrino sector. Conclusions and remarks can be found in sect. 5.

2 The models

The general Yukawa Lagrangian is given in eq.(1). To have the actual form of the mass matrices, one must give the Yukawa coupling constants Y_{ij}^{10} and $Y_{ij}^{\overline{126}}$ and the VEVs of the Higgs representations ϕ_{10}^{ij} and $\phi_{\overline{126}}^{ij}$. The discrete symmetries, to be discussed later, will play here an important role. Those symmetries fix the non-zero entries of the Yukawa matrices and ensure the stability of our predictions.

Let us, however, discuss first the possible entries to the matrices on a pure phenomenological level. Both ϕ_{10} and $\phi_{\overline{126}}$ can develop VEVs in the directions of the down and/or the up quarks. However, only the $\phi_{\overline{126}}$ multiplets allow for a B–L violating VEV, which generates the Majorana mass matrix of the RH- neutrino. We assume, as usual, that below the GUT scale one has effectively the MSSM with two doublets of Higgs H_u and H_d . These are mixtures of the SM doublet components of all scalar SO(10) representations, also those needed for the local symmetry breaking. We can therefore separate the Yukawa terms into five groups, even at the SO(10) GUT scale as follows:

$$\begin{aligned} \mathcal{L}_Y = & \sum Y_{ij} \left\{ a^{ij} \left[(\overline{d}_{Ri} d_{Lj} + \overline{\ell}_{Ri} \ell_{Lj}) H_{10,d}^{ij} + (\overline{u}_{Ri} u_{Lj} + \overline{\nu}_{Ri} \nu_{Lj}) H_{10,u}^{ij} \right] \right. \\ & + b^{ij} \left[(\overline{d}_{Ri} d_{Lj} - 3\overline{\ell}_{Ri} \ell_{Lj}) H_{126,d}^{ij} + (\overline{u}_{Ri} u_{Lj} - 3\overline{\nu}_{Ri} \nu_{Lj}) H_{126,u}^{ij} \right. \\ & \left. \left. + \overline{\nu}_{Ri} \nu_{Rj} (-3) \phi_{\overline{126},1_{SU(5)}}^{ij} \right] \right\}. \end{aligned} \quad (2)$$

In view of the requirement (2) that only one of the ϕ_{10}^{ij} or $\phi_{\overline{126}}^{ij}$ can contribute to the mass matrices, we have for the non-vanishing Yukawa matrix elements only one of the two possibilities:

$$(a^{ij}, b^{ij}) = (1, 0) \quad \text{or} \quad (0, 1).$$

The quark mass matrices develop below the GUT scale, in terms of the MSSM,

the following contributions, which define the effective Yukawa matrices used in the RGEs:

$$(M_d)_{ij} = Y_{ij}(a_{ij}\gamma_d^{ij} + b_{ij}\theta_d^{ij}) \cos \beta v \quad (3)$$

$$(M_u)_{ij} = Y_{ij}(a_{ij}\gamma_u^{ij} + b_{ij}\theta_u^{ij}) \sin \beta v \quad (4)$$

where, γ^{ij} and θ^{ij} account for the amount of mixing of the VEVs of the MSSM doublets $\langle H_d \rangle$ and $\langle H_u \rangle$. Also, as usual in the MSSM:

$$\tan \beta = \frac{\langle H_u \rangle}{\langle H_d \rangle} \quad \text{and} \quad v = \sqrt{\langle H_u \rangle^2 + \langle H_d \rangle^2} = 174 \text{ GeV}.$$

Now, for the phenomenological “good” textures, there are additional restrictions: The Yukawa couplings Y_{ij} vanish when the corresponding texture zeros are common to both M_d and M_u . E.g. $Y_{ij} = 0$ in all texture sets. For zero entries in only one matrix we have:

$$a_{ij}\gamma_u^{ij} + b_{ij}\theta_u^{ij} = 0 \quad \text{or} \quad a_{ij}\gamma_d^{ij} + b_{ij}\theta_d^{ij} = 0.$$

As for the non-vanishing (i, j) matrix elements – it is impossible to say which Higgs representation $\phi_{\mathbf{10}}^{ij}$ or $\phi_{\mathbf{\overline{126}}}^{ij}$ contributes, as long as only the quark masses and mixing angles are used.

Our phenomenological discussion is based on the five sets of texture zeros for the quarks, given in table 1.

Now, to predict the neutrino matrices we must know which of the Higgs representations, $\phi_{\mathbf{10}}$ or $\phi_{\mathbf{\overline{126}}}$, contributes to the different matrix elements. As was already discussed in the introduction, the structure of M_d and M_ℓ is fixed by the need to have the approximate Yukawa unification and the Georgi-Jarlskog mass relations. The result is that all the non-vanishing matrix elements will be generated by $\phi_{\mathbf{10}}$, except for the (2,2) one which is due to $\phi_{\mathbf{\overline{126}}}$. (I.e. it obtains a relative factor (-3) in $(M_\ell)_{22}$ relative to $(M_d)_{22}$).

The explicit structure of those matrices, for the different textures, can be found in table 3.

In order to fix the structure of M_u we must go in a different direction, as M_{ν_D} is unknown phenomenology. We shall use our predictability and minimality requirements to restrict considerably the number of possibilities. The resulting M_u textures will

d dictate the neutrino matrices.

The arguments go as follows:

All non-vanishing entries to M_{ν_R} must be generated by one $\phi_{\overline{126}}$ and this should be induced via our discrete symmetries. Those, however, allow for one Higgs multiplet to couple to at most two (i, j) entries. Hence, only the following non-singular possibilities are open:

$$M_{\nu_R}^I = \begin{pmatrix} 0 & y & 0 \\ y & 0 & 0 \\ 0 & 0 & x \end{pmatrix}, \quad M_{\nu_R}^{II} = \begin{pmatrix} 0 & 0 & y \\ 0 & x & 0 \\ y & 0 & 0 \end{pmatrix}$$

and

$$M_{\nu_R}^{III} = \begin{pmatrix} x & 0 & 0 \\ 0 & 0 & y \\ 0 & y & 0 \end{pmatrix}.$$

The two last possibilities, however, cannot be realized in our textures. For $M_{\nu_R}^{III}$ it is clear because in all the five quark texture $Y_{11} = 0$. For $M_{\nu_R}^{II}$ it is more complicated. If one Higgs representation induces contributions to two entries (i, j) and (k, l) in one of the matrices, it means that $\bar{\Psi}_i^c \Psi_j$ and $\bar{\Psi}_k^c \Psi_l$ have the same quantum numbers. Thus, *all* mass matrices acquire a contribution in *both* entries or *no one at all*. (In the last case, all Higgs representations with the above quantum number do not develop a VEV in the relevant direction). In our case $(M_d)_{22} \neq 0$ while $(M_d)_{13} = (M_d)_{31} = 0$ in all textures and $M_{\nu_R}^{II}$ is inconsistent with the above requirement.

Now, $\phi_{\overline{126}}$ which generates $M_{\nu_R}^I$ can contribute to M_u and M_{ν_R} only, as $(M_d)_{33}$ must come from a ϕ_{10} . Hence, the $\phi_{\overline{126}}$ representation which induces the Majorana M_{ν_R} can contribute to M_u and M_{ν_D} only. Minimality then requires that this must be the case.

One finds, by explicit observation, that $M_{\nu_R}^I$ is relevant for the texture sets 1,2 and 4. This fixes three matrix elements in M_u . The other entries required by the five quark textures can get contributions from both ϕ_{10} or $\phi_{\overline{126}}$. M_{ν_D} is then obtained from M_u using suitable Clebsch-Gordan factors. Note, that the magnitude of those contributions is given by the phenomenology i.e by fitting the evolved M_u matrix to the observed masses and mixing angles. Only the factors accompanying these contributions in M_{ν_D} are dictated by the choice of ϕ_{10} or $\phi_{\overline{126}}$.

We have, therefore, several possible combinations for each texture set and in total eight different ones. Those are presented in table 3.

Once the neutrino matrices M_{ν_D} and M_{ν_R} are known, we can construct the see-saw matrix for each model, in the form:

$$M_\nu^{light} \simeq -M_{\nu_D} M_{\nu_R}^{-1} M_{\nu_D}. \quad (5)$$

After this matrix is evolved to low energies (see next section) it gives us the neutrino masses and mixing angles. To calculate the mixing angles one must obviously consider the charged lepton mass matrix as well. The angles are, however, independent on the overall mass scale of the RH-neutrinos. The latter is a free parameter in our models and hence, we predict the neutrino mass ratios only. The RH-neutrino scale will be fixed latter for models with mixing angles which allow for a solution to the SNP, such that $\Delta m_{e\mu}^2$ have the right value.

We know now phenomenologically what the different textures are and it remains only to show how those textures can be induced using discrete symmetries. This is actually strait forward and very similar in the different models.

Let the fermions and Higgs representation have the following transformation properties under our symmetry:

$$\begin{aligned} \psi_j &\rightarrow e^{i\alpha_j} \psi_j \quad \text{and} \quad \phi_j^{\mathbf{10}} \rightarrow e^{i\beta_j} \phi_j^{\mathbf{10}} \\ \phi_j^{\overline{\mathbf{126}}} &\rightarrow e^{i\gamma_j} \phi_j^{\overline{\mathbf{126}}}. \end{aligned}$$

We must require that $(M_u)_{12}$ and $(M_d)_{33}$ are generated by one $\phi_{\overline{\mathbf{126}}}$. Hence,

$$\alpha_1 + \alpha_2 = 2\alpha_3 = -\gamma_1.$$

However, M_d gets also contributions at the (1,2) and (3,3) entries, via $\phi_{\mathbf{10}}$. As our symmetry is on the SO(10) level, those matrix elements also, must be due to the same Higgs representation i.e $\phi_{\mathbf{10}}$ in this case and

$$\beta_1 = \gamma_1.$$

This means that $\phi_{\overline{\mathbf{126}}}^1$ generates a light VEV in the u-direction while $\phi_{\mathbf{10}}^1$ generates one in the d-direction. The other entries acquire contributions according to the corresponding quantum numbers.

As an explicit realization we can take:

$$\alpha_1 = 1, \quad \alpha_2 = 3 \quad \text{and} \quad \alpha_3 = 2.$$

in this case:

$$\beta_1 = \gamma_1 = -4.$$

E.g for the texture 1_I we have , in addition:

$$\gamma_2(\phi_{\mathbf{10}}^{22}) = \beta_2(\phi_{\overline{\mathbf{126}}}^{22}) = -2\alpha_2 = -6$$

and

$$\beta_3(\phi_{\mathbf{10}}^{23}) = -(\alpha_2 + \alpha_3) = -5.$$

So finally for this texture we need:

$$3 \times \phi_{\mathbf{10}} \quad \text{and} \quad 2 \times \phi_{\overline{\mathbf{126}}} ,$$

of which only $\phi_{\overline{\mathbf{126}}}^1$ generates a heavy VEV.

In all other models very similar discrete symmetries are needed.

3 Renormalization Group Equations and Fits

All the matrix elements of our matrices are in principle complex numbers. One can, however, use the freedom to redefine the nine phases of the three LH-doublets and six RH-singlets of the SM, to reduce considerably the number of the “physical” phases. In any case, symmetric quark matrices can be always transformed into hermitian ones in this way [6]. As we are interested only in the neutrino sector and the leptonic phases cannot be observed - we use for simplicity only one physical phase. Let us put it at the (1,2) matrix element and in an hermitian way - as DHR [8] do.

As an example, we give in the following the explicit matrix elements of the model discussed in the previous section.

Model 1_I :

$$\mathbf{Y}_U = \begin{pmatrix} 0 & C_u & 0 \\ C_u & B_u & 0 \\ 0 & 0 & A_u \end{pmatrix} \quad \mathbf{Y}_D = \begin{pmatrix} 0 & D_d e^{i\phi} & 0 \\ D_d e^{-i\phi} & C_d & B_d \\ 0 & B_d & A_d \end{pmatrix} \quad (6)$$

$$\mathbf{Y}_{\nu_D} = \begin{pmatrix} 0 & -3C_u & 0 \\ -3C_u & B_u & 0 \\ 0 & 0 & -3A_u \end{pmatrix} \quad \mathbf{Y}_L = \begin{pmatrix} 0 & D_d e^{i\phi} & 0 \\ D_d e^{-i\phi} & -3C_d & B_d \\ 0 & B_d & A_d \end{pmatrix} \quad (7)$$

$$\mathbf{Y}_{\nu_M} = \begin{pmatrix} 0 & C_u & 0 \\ C_u & 0 & 0 \\ 0 & 0 & A_u \end{pmatrix} \quad (8)$$

In order to extract the neutrino properties from these matrices, the matrix elements A_u , B_u , ..., D_d and ϕ have to be determined. We do this, as usual, by fitting masses and mixing angles as predicted by the textures to their experimental values. Since these specific textures for the Yukawa matrices are given at the unification scale M_X , we must evolve the matrices from the GUT scale, M_X to low energies using the renormalization group equations (RGEs) (see Appendix A.1).

In our model, the SUSY-SO(10) is broken at M_X directly into the MSSM. The MSSM is broken at the effective $M_{SUSY} \approx 100\text{GeV}$ into the SM which is broken in its turn effectively at M_Z into $SU_C(3) \times U_{EM}(1)$. We take, as it is done in many papers, $M_{SUSY} = M_Z$. A different choice will have only a minor effect on the neutrino properties.

The renormalization group equations for the gauge and Yukawa coupling constants [19] are coupled, non-linear first order differential equations, which do not have a complete analytical solution. We therefore use a numerical procedure both to solve the RGEs and to fit the Yukawa matrix parameters.

For the Yukawa RGEs we use the effective Yukawa matrices $\Lambda^{u,d,\dots}$ defined by

$$M_u = \Lambda^u v \sin \beta \quad , \quad M_d = \Lambda^d v \cos \beta \quad e.t.c. \quad (9)$$

using equations (3) and (4). Thus, for a given value of $\tan \beta$, Λ^i are obtained in terms of the mass matrices. The explicit calculations were done using the semi-analytic form due to Barger, Berger and Ohmann [20] (see Appendix A.1). This form reduces the number of variables significantly. To fix the parameters of a given texture, we obtained first the masses and mixing angles as functions of the GUT scale parameters and then evolved them to low energies. Those are then fitted to the experimental values using the 'shooting' method [21] (see Appendix A.2).

The run of the Yukawa and gauge coupling constants from M_X down to M_Z is done in terms of the two loop RGEs of the MSSM [19]. The appropriate boundary conditions for the Yukawa and gauge couplings are applied at M_Z . Below M_Z , three loop QCD and one loop QED renormalization group equations, are used. We compare then our parameters with the standard masses of the light quarks and charged leptons given at $\mu = 1\text{GeV}$, and the heavy quarks at their physical masses (see Table 2).

Since all of our textures have eight parameters, we have to fit to eight experimental quantities. We use, out of the experimental data displayed in Table 2 m_b , m_c , m_u , m_e , m_μ , m_τ , $|V_{us}|$ and $|V_{cb}|$ to fix the texture parameters. In addition, we take $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$ to fix the GUT scale M_X and the gauge coupling $\alpha(M_X)$ at the GUT scale.

Now, the texture parameters found with the shooting procedure define the see-saw

matrix (5). We evolve this matrix from M_X to M_Z using one-loop renormalization group equations [22] (see Appendix A.3).

As a result we obtain, for each texture, a set of solutions which give a good fit to the data, i.e $\chi^2 < 1$. Those solutions are parametrized according to the value of $\tan \beta$. One of the predicted parameters is m_t . The dependence of m_t on $\tan \beta$ is given in Fig. 1, for the texture 1_I . For all other textures it is practically the same, as we have in all our models the approximate $\tau - b$ Yukawa unification. One sees, as it is already well known in this case, that small and large values of $\tan \beta$ are preferred. As those correspond to two different physical situations [25], we will present our results for $\tan \beta = 1.5$ and $\tan \beta = 55$.

4 Discussion of the results.

Our results for the different textures are displayed in table 4.

Looking at this table one sees clearly that we do not have large mixing angles. Practically speaking, all our solutions obey

$$\sin^2 2\theta < 0.2 .$$

This means that our models cannot allow for the depletion of the atmospheric muon neutrinos [11]. Also, vacuum oscillations will not be able to serve as a solution to the SNP [10] and only the small angle (i.e adiabatic) MSW mechanism can work.

Using the recent estimate for the small angle MSW region [26]:

$$\sin^2 2\theta_{e\mu} = 6 \times 10^{-4} - 2 \times 10^{-2},$$

one sees that the models 1_{II} , 4_{III} , 4_{IV} , can explain the SNP. This is obviously provided⁵:

$$\Delta m_{e\mu}^2 \simeq 4 \times 10^{-5}.$$

This requirement fixes the RH-neutrino scale to be:

$$M_R = 10^{13} - 10^{14} GeV.$$

Knowing the neutrino mass ratios we can compute the corresponding masses of the τ -neutrinos. Those are found to be all in the few eV region i.e. interesting for cosmology [12]. Also, the corresponding $\nu_\tau - \nu_\mu$ mixing angles are large enough for 1_{II} and 4_{IV} to be observed in the already running CERN CHORUS [16] and NOMAD [17] experiments, as well as in the approved FERMILAB P803 [18] one. Also, some of the models which cannot solve the SNP have relatively large $\sin^2 2\theta_{\mu\tau}$ which can be observed in the above experiments.

⁵Note, that this the mass difference relevant for our mixing angles in the range $\sin^2 2\theta_{e\mu} \simeq (1-2) \times 10^{-2}$.

5 Conclusions and remarks

We looked in this paper for SUSY-SO(10) models which can predict the neutrino-spectrum in terms of the “known” parameters of the charged fermions. The main idea is to dictate the mass matrix of the RH-neutrinos rather than simply conjecture its form, as it is done in many models. To do this we used the requirements of DHR and suitable discrete symmetries. Starting then from the general classification of “good” symmetric textures for the quark mass matrices by RRR, we predicted correspondingly eight neutrino mass matrices at the GUT scale. In evolving these mass matrices to low energies we made some approximations: a)we neglected threshold effects at the GUT scale as well as at M_{SUSY} which we took to be M_Z . b)we started the renormalization of the see-saw matrix also from the GUT scale and not from M_R . c)we made a simplifying conjecture for the unobservable leptonic phases. Those approximations, however, cannot change the qualitative predictions of our models. They can at most change somewhat the neutrino mixing angles. Practically speaking, one must allow for up to 10% deviations from our predictions of the neutrino properties.

We also required that our SUSY-SO(10) is “the whole story”. I.e. we did not use possible non-renormalizable effective contributions due to physics at the Planck scale (like gravity or superstrings). Such contributions are frequently used in recent papers. There are very many possible contributions of which one picks up those suitable for his arguments and neglects arbitrarily all others. Such a procedure destroys the predictability which is the main ingredient of our models. Also, one can imagine scenarios where the non-renormalizable effects are negligible and that we actually assume.

There is, however, one indirect evidence that physics at the Planck-mass may be relevant to our models. This is related to the RH-neutrino mass-scale which is a free parameter . Yet, to explain the solar neutrino puzzle and get τ -neutrino masses relevant for cosmology, we need $M_R = 10^{13} - 10^{14} \text{GeV}$ which is equal to $\frac{M_{GUT}^2}{M_{Planck}}$. It is also interesting to embed such an intermediate scale into the local symmetry breaking of SUSY-SO(10), in order to make it natural[27].

A Appendix

A.1 Semi-analytic approach

In the renormalization group equations for the Yukawa couplings the largest Yukawa contributions come from the Yukawa couplings of the third generation y_t , y_b and y_τ . In view of this fact, Barger et. al. [20] find the following RGEs for the Yukawa couplings:

$$\begin{aligned} \frac{d\lambda_i}{dt} &= \frac{\lambda_i}{16\pi^2} \left[x_1 + x_2 \lambda_i^2 + a_u \sum_{\alpha} \lambda_{\alpha}^2 |V_{i\alpha}|^2 \right. \\ &\quad \left. + \frac{1}{16\pi^2} \left(x_3 + x_4 \lambda_i^2 + x_5 \lambda_i^4 + \sum_{\alpha} (b_u \lambda_{\alpha}^2 + c_u \lambda_{\alpha}^4 + (d_u + e_u) \lambda_i^2 \lambda_{\alpha}^2) |V_{i\alpha}|^2 \right) \right], \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{d\lambda_{\alpha}}{dt} &= \frac{\lambda_{\alpha}}{16\pi^2} \left[x_6 + x_7 \lambda_{\alpha}^2 + a_d \sum_i \lambda_i^2 |V_{i\alpha}|^2 \right. \\ &\quad \left. + \frac{1}{16\pi^2} \left(x_8 + x_9 \lambda_{\alpha}^2 + x_{10} \lambda_{\alpha}^4 + \sum_i (b_d \lambda_i^2 + c_d \lambda_i^4 + (d_d + e_d) \lambda_{\alpha}^2 \lambda_i^2) |V_{i\alpha}|^2 \right) \right], \end{aligned} \quad (11)$$

$$\frac{d\lambda_a}{dt} = \frac{\lambda_a}{16\pi^2} \left[x_{11} + x_{12} \lambda_a^2 + \frac{1}{16\pi^2} \left(x_{13} + x_{14} \lambda_a^2 + x_{15} \lambda_a^4 \right) \right], \quad (12)$$

where $i = u, c, t$, $\alpha = d, s, b$ and $a = e, \mu, \tau$. The CKM matrix elements $W_1 = |V_{cb}|^2$, $|V_{ub}|^2$, $|V_{ts}|^2$, $|V_{td}|^2$, J evolve according to

$$\frac{dW_1}{dt} = -\frac{W_1}{8\pi^2} \left[(a_d \hat{\lambda}_t^2 + a_u \hat{\lambda}_b^2) + \frac{1}{(16\pi^2)} (e_d + e_u) \lambda_t^2 \lambda_b^2 \right], \quad (13)$$

with

$$\hat{\lambda}_b^2 = \lambda_b^2 \left(1 + \frac{b_u + c_u \lambda_b^2}{16\pi^2 a_u} \right), \quad (14)$$

$$\hat{\lambda}_t^2 = \lambda_t^2 \left(1 + \frac{b_d + c_d \lambda_t^2}{16\pi^2 a_d} \right). \quad (15)$$

For $W_2 = |V_{us}|^2, |V_{cd}|^2, |V_{tb}|^2, |V_{cs}|^2, |V_{ud}|^2$ we have

$$\frac{dW_2}{dt} = 0. \quad (16)$$

The various coefficients for the RGEs in the MSSM are given in table 5.

A.2 Determining the texture parameters

To determine the texture parameters, we fit the low energy predictions for masses and mixing angles to the experimental values. In our case, all textures have eight

parameters, so we fit to eight experimentally known quantities, namely m_b , m_c , m_u , m_e , m_μ , m_τ , $|V_{us}|$ and $|V_{cb}|$. The procedure employed here is called ‘shooting’.

Let p_j be the texture parameters and $R_i(p_j)$ the low energy predictions obtained by the above mentioned running procedure. Further, let r_i denote their experimental values. We then have to solve the system of equations

$$R_i(p_j) - r_i = 0. \quad (17)$$

This is done by an iterative numerical procedure.

A.3 Renormalization of the see-saw matrix

Following the authors of [22], we define the neutrino see-saw mass coefficient

$$\frac{1}{2}c_1^{ab} = c_{21}^{ab} = c_{22}^{ab} = \frac{1}{2}c_3^{ab} = Y_l^{ca}(M_R^{-1})^{cd}Y_l^{db}. \quad (18)$$

The renormalization group equations for the see-saw matrix in the strict SUSY limit are:

$$\begin{aligned} \frac{d}{dt}c_1^{ab} &= \frac{1}{16\pi^2} \left(\left[\frac{1}{2}2g_1^2 + 2g_2^2 + 6 \operatorname{tr}(Y_u Y_u^\dagger) \right] c_1^{ab} + (Y_l Y_l^\dagger)^{bc} c_1^{ca} + (Y_l Y_l^\dagger)^{ac} c_1^{cb} \right. \\ &\quad \left. - (2g_1^2 + 6g_2^2) (c_{21}^{ab} + c_{21}^{ba}) - (2g_1^2 + 6g_2^2) (c_{22}^{ab} + c_{22}^{ba}) \right) \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{d}{dt}c_3^{ab} &= \frac{1}{16\pi^2} \left(\left[2g_1^2 + 2g_2^2 + 6 \operatorname{tr}(Y_u Y_u^\dagger) \right] c_3^{ab} + (Y_l Y_l^\dagger)^{bc} c_3^{ca} + (Y_l Y_l^\dagger)^{ac} c_3^{cb} \right. \\ &\quad \left. - (2g_1^2 + 6g_2^2) (c_{21}^{ab} + c_{21}^{ba}) - (2g_1^2 + 2g_2^2) (c_{22}^{ab} + c_{22}^{ba}) \right) \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{d}{dt}c_{21}^{ab} &= \frac{1}{16\pi^2} \left(\left[4g_2^2 - 2g_1^2 + 6 \operatorname{tr}(Y_u Y_u^\dagger) \right] c_{21}^{ab} + 2g_2^2 c_{21}^{ba} + (Y_l Y_l^\dagger)^{bc} c_{21}^{ca} + (Y_l Y_l^\dagger)^{ac} c_{21}^{cb} \right. \\ &\quad \left. + (g_1^2 - g_2^2) (c_{22}^{ab} + c_{22}^{ba}) - \frac{1}{2} (g_1^2 + 5g_2^2) (c_1^{ab} + c_3^{ab}) \right) \end{aligned} \quad (21)$$

$$\begin{aligned} \frac{d}{dt}c_{22}^{ab} &= \frac{1}{16\pi^2} \left(\left[-4g_1^2 - 2g_1^2 + 6 \operatorname{tr}(Y_u Y_u^\dagger) \right] c_{22}^{ab} + 2g_2^2 c_{22}^{ba} + (Y_l Y_l^\dagger)^{bc} c_{22}^{ca} + (Y_l Y_l^\dagger)^{ac} c_{22}^{cb} \right. \\ &\quad \left. + (g_1^2 - g_2^2) (c_{21}^{ab} + c_{21}^{ba}) - 4g_2^2 c_{21}^{ab} - \frac{1}{2} (g_1^2 - g_2^2) (c_1^{ab} + c_3^{ab}) \right) \end{aligned} \quad (22)$$

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Tables

Texture	Y_u	Y_d
1	$\begin{pmatrix} 0 & C & 0 \\ C & B & 0 \\ 0 & 0 & A \end{pmatrix}$	$\begin{pmatrix} 0 & F & 0 \\ F^* & E & E' \\ 0 & E' & D \end{pmatrix}$
2	$\begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}$	$\begin{pmatrix} 0 & F & 0 \\ F^* & E & E' \\ 0 & E'^* & D \end{pmatrix}$
3	$\begin{pmatrix} 0 & 0 & C \\ 0 & B & 0 \\ C & 0 & A \end{pmatrix}$	$\begin{pmatrix} 0 & F & 0 \\ F^* & E & E' \\ 0 & E' & D \end{pmatrix}$
4	$\begin{pmatrix} 0 & C & 0 \\ C & B & B' \\ 0 & B' & A \end{pmatrix}$	$\begin{pmatrix} 0 & F & 0 \\ F^* & E & 0 \\ 0 & 0 & D \end{pmatrix}$
5	$\begin{pmatrix} 0 & 0 & C \\ 0 & B & B' \\ C & B' & A \end{pmatrix}$	$\begin{pmatrix} 0 & F & 0 \\ F^* & E & 0 \\ 0 & 0 & D \end{pmatrix}$

Table 1: The five sets of symmetric quark mass matrices with texture zeros found by RRR [6]

Gauge couplings [21, 24]	Quark masses [23, 24]
$\alpha_1(M_Z) = 0.01698 \pm 0.00009$	$m_u(1 \text{ GeV}) = 5.1 \pm 1.5 \text{ MeV}$
$\alpha_2(M_Z) = 0.03364 \pm 0.0002$	$m_d(1 \text{ GeV}) = 8.9 \pm 2.6 \text{ MeV}$
$\alpha_3(M_Z) = 0.120 \pm 0.007 \pm 0.002$	$m_s(1 \text{ GeV}) = 175 \pm 55 \text{ MeV}$
	$m_c(m_c) = 1.27 \pm 0.05 \text{ GeV}$
	$m_b(m_b) = 4.4 \pm 0.10 \text{ GeV}$
Lepton masses [21]	CKM matrix entries [24]
$m_e(1 \text{ GeV}) = 0.496 \text{ MeV}$	$ V_{us} = 0.218 - 0.224$
$m_\mu(1 \text{ GeV}) = 104.57 \text{ MeV}$	$ V_{ub} = 0.002 - 0.005$
$m_\tau(1 \text{ GeV}) = 1.7835 \text{ GeV}$	$ V_{cb} = 0.032 - 0.048$

Table 2: Experimental data used to fix the GUT scale parameters

Texture 1	U	D
I	$\mathbf{U} = \begin{pmatrix} 0 & \mathbf{126}_1 & 0 \\ \mathbf{126}_1 & \mathbf{10}_2 & 0 \\ 0 & 0 & \mathbf{126}_1 \end{pmatrix}$	$\mathbf{D} = \begin{pmatrix} 0 & \mathbf{10}_1 & 0 \\ \mathbf{10}_1 & \mathbf{126}_2 & \mathbf{10}_3 \\ 0 & \mathbf{10}_3 & \mathbf{10}_1 \end{pmatrix}$
II	$\mathbf{U} = \begin{pmatrix} 0 & \mathbf{126}_1 & 0 \\ \mathbf{126}_1 & \mathbf{126}_2 & 0 \\ 0 & 0 & \mathbf{126}_1 \end{pmatrix}$	$\mathbf{D} = \begin{pmatrix} 0 & \mathbf{10}_1 & 0 \\ \mathbf{10}_1 & \mathbf{126}_2 & \mathbf{10}_2 \\ 0 & \mathbf{10}_2 & \mathbf{10}_1 \end{pmatrix}$
Texture 2		
I	$\mathbf{U} = \begin{pmatrix} 0 & \mathbf{126}_1 & 0 \\ \mathbf{126}_1 & 0 & \mathbf{10}_2 \\ 0 & \mathbf{10}_2 & \mathbf{126}_1 \end{pmatrix}$	$\mathbf{D} = \begin{pmatrix} 0 & \mathbf{10}_1 & 0 \\ \mathbf{10}_1 & \mathbf{126}_2 & \mathbf{10}_2 \\ 0 & \mathbf{10}_2 & \mathbf{10}_1 \end{pmatrix}$
II	$\mathbf{U} = \begin{pmatrix} 0 & \mathbf{126}_1 & 0 \\ \mathbf{126}_1 & 0 & \mathbf{126}_3 \\ 0 & \mathbf{126}_3 & \mathbf{126}_1 \end{pmatrix}$	$\mathbf{D} = \begin{pmatrix} 0 & \mathbf{10}_1 & 0 \\ \mathbf{10}_1 & \mathbf{126}_2 & \mathbf{10}_2 \\ 0 & \mathbf{10}_2 & \mathbf{10}_1 \end{pmatrix}$
Texture 4	U	D
I	$\mathbf{U} = \begin{pmatrix} 0 & \mathbf{126}_1 & 0 \\ \mathbf{126}_1 & \mathbf{10}_2 & \mathbf{10}_3 \\ 0 & \mathbf{10}_3 & \mathbf{126}_1 \end{pmatrix}$	$\mathbf{D} = \begin{pmatrix} 0 & \mathbf{10}_1 & 0 \\ \mathbf{10}_1 & \mathbf{126}_2 & 0 \\ 0 & 0 & \mathbf{10}_1 \end{pmatrix}$
II	$\mathbf{U} = \begin{pmatrix} 0 & \mathbf{126}_1 & 0 \\ \mathbf{126}_1 & \mathbf{126}_2 & \mathbf{10}_2 \\ 0 & \mathbf{10}_2 & \mathbf{126}_1 \end{pmatrix}$	$\mathbf{D} = \begin{pmatrix} 0 & \mathbf{10}_1 & 0 \\ \mathbf{10}_1 & \mathbf{126}_2 & 0 \\ 0 & 0 & \mathbf{10}_1 \end{pmatrix}$
III	$\mathbf{U} = \begin{pmatrix} 0 & \mathbf{126}_1 & 0 \\ \mathbf{126}_1 & \mathbf{10}_2 & \mathbf{126}_3 \\ 0 & \mathbf{126}_3 & \mathbf{126}_1 \end{pmatrix}$	$\mathbf{D} = \begin{pmatrix} 0 & \mathbf{10}_1 & 0 \\ \mathbf{10}_1 & \mathbf{126}_2 & 0 \\ 0 & 0 & \mathbf{10}_1 \end{pmatrix}$
IV	$\mathbf{U} = \begin{pmatrix} 0 & \mathbf{126}_1 & 0 \\ \mathbf{126}_1 & \mathbf{126}_2 & \mathbf{126}_3 \\ 0 & \mathbf{126}_3 & \mathbf{126}_1 \end{pmatrix}$	$\mathbf{D} = \begin{pmatrix} 0 & \mathbf{10}_1 & 0 \\ \mathbf{10}_1 & \mathbf{126}_2 & 0 \\ 0 & 0 & \mathbf{10}_1 \end{pmatrix}$

Table 3: Explicit structure of the eight “good” textures

$\tan \beta$	$\sin^2 2\theta_{e\mu}$	$\sin^2 2\theta_{\mu\tau}$	m_{ν_μ}/m_{ν_e}	m_{ν_τ}/m_{ν_μ}	m_{ν_τ} [eV]
1_I					
1.5	5.6×10^{-2}	2.8×10^{-3}	124	3100	6.2
55	5.2×10^{-2}	4.5×10^{-3}	124	1900	3.8
1_{II}					
1.5	2.0×10^{-2}	2.8×10^{-3}	1100	1040	2.1
55	2.1×10^{-2}	4.5×10^{-3}	1100	650	1.3
2_I					
1.5	1.9×10^{-1}	2.8×10^{-2}	33	6050	12
55	2.0×10^{-1}	5.4×10^{-2}	33	3750	7.5
2_{II}					
1.5	2.8×10^{-2}	6.5×10^{-4}	2470	700	1.4
55	3.1×10^{-2}	1.7×10^{-3}	2460	435	0.87
4_I					
1.5	3.6×10^{-2}	1.4×10^{-3}	434	1650	3.3
55	3.9×10^{-2}	2.2×10^{-3}	350	1150	2.3
4_{II}					
1.5	2.4×10^{-2}	1.3×10^{-2}	2180	740	1.5
55	2.6×10^{-2}	2.0×10^{-2}	1500	555	1.1
4_{III}					
1.5	1.7×10^{-2}	1.5×10^{-3}	2440	700	1.4
55	1.7×10^{-2}	2.3×10^{-3}	2100	470	0.94
4_{IV}					
1.5	1.9×10^{-2}	1.3×10^{-2}	550	1480	3.0
55	1.8×10^{-2}	2.0×10^{-2}	655	840	1.68

Table 4: The neutrino properties predicted by the “good” textures

up quarks	down quarks	charged leptons
$a_u = 1$	$a_d = 1$	
$b_u = \frac{2}{5}g_1^2 - (3\lambda_b^2 + \lambda_\tau^2)$	$b_d = \frac{4}{5}g_1^2 - 3\lambda_t^2$	
$c_u = -2$	$c_d = -2$	
$d_u = -2$	$d_d = -2$	
$e_u = 0$	$e_d = 0$	
$x_1 = 3\lambda_t^2$ - $\left(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)$	$x_6 = 3\lambda_b^2 + \lambda_\tau^2$ - $\left(\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)$	$x_{11} = 3\lambda_b^2 + \lambda_\tau^2$ - $\left(\frac{9}{5}g_1^2 + 3g_2^2\right)$
$x_2 = 3$	$x_7 = 3$	$x_{12} = 3$
$x_3 = -9y_t^4 - 3y_t^2y_b^2$ + $y_t^2(\frac{4}{5}g_1^2 + 16g_3^2)$ + $\left(\frac{13}{15}(2n_g + \frac{3}{5}) + \frac{169}{450}\right)g_1^4$ + $\left(3(2n_g - 5) + \frac{9}{2}\right)g_2^4$ + $\left(\frac{16}{3}(2n_g - 9) + \frac{128}{9}\right)g_3^4$ + $g_1^2g_2^2 + \frac{136}{45}g_1^2g_3^2 + 8g_2^2g_3^2$	$x_8 = -9y_b^4 - 3y_t^2y_b^2 - 3y_\tau^4$ + $y_b^2(-\frac{2}{5}g_1^2 + 16g_3^2) + y_\tau^2(\frac{6}{5}g_1^2)$ + $\left(\frac{7}{15}(2n_g + \frac{3}{5}) + \frac{49}{450}\right)g_1^4$ + $\left(3(2n_g - 5) + \frac{9}{2}\right)g_2^4$ + $\left(\frac{16}{3}(2n_g - 9) + \frac{128}{9}\right)g_3^4$ + $g_1^2g_2^2 + \frac{8}{9}g_1^2g_3^2 + 8g_2^2g_3^2$	$x_{13} = -9y_b^4 - 3y_t^2y_b^2 - 3y_\tau^4$ + $y_b^2(-\frac{2}{5}g_1^2 + 16g_3^2) + y_\tau^2(\frac{6}{5}g_1^2)$ + $\left(\frac{9}{5}(2n_g + \frac{3}{5}) + \frac{81}{50}\right)g_1^4$ + $\left(3(2n_g - 5) + \frac{9}{2}\right)g_2^4$ + $\frac{9}{5}g_1^2g_2^2$
$x_4 = \frac{2}{5}g_1^2 + 6g_2^2 - 9y_t^2$	$x_9 = \frac{4}{5}g_1^2 + 6g_2^2 - 9y_b^2 - 3y_\tau^2$	$x_{14} = 6g_2^2 - 9y_b^2 - 3y_\tau^2$
$x_5 = -4$	$x_{10} = -4$	$x_{15} = -4$

Table 5: RGE coefficients for the MSSM

FIGURE CAPTION

Fig. I: The dependence of m_t on $\tan \beta$ for the texture 1_I .

Figure 1

